Test Review
5.5–6.2 (Key)

1. $\alpha = 28°$, $\beta = 46°$, $c = 17$

$\gamma = 180° - 28° - 46° = \boxed{106°}$

$\dfrac{\sin 106°}{17} = \dfrac{\sin 28°}{a}$ $\quad a = \dfrac{17 \sin 28°}{\sin 106°} = \boxed{8.3}$

$\dfrac{\sin 106°}{17} = \dfrac{\sin 46°}{b}$ $\quad b = \dfrac{17 \sin 46°}{\sin 106°} = \boxed{12.7}$

2. $\alpha = 41.2°$, $a = 8.1$, $b = 10.6$

$b > a > b \sin \alpha$

$10.6 > 8.1 > 10.6 \sin 41.2° = 7$

so, 2 - triangles possible.

1st Triangle

$\dfrac{\sin 41.2°}{8.1} = \dfrac{\sin \beta_1}{10.6}$ $\quad \beta_1 = \sin^{-1}\left(\dfrac{10.6 \sin 41.2°}{8.1}\right) = \boxed{59.5°}$

$\gamma_1 = 180° - 41.2° - 59.5° = \boxed{79.3°}$ $\quad \dfrac{\sin 79.3°}{c_1} = \dfrac{\sin 41.2°}{8.1}$

$c_1 = \dfrac{8.1 \sin 79.3°}{\sin 41.2°} = \boxed{12.1}$

2nd Triangle

$\beta_2 = 180° - \beta_1 \Rightarrow 180° - 59.5° = \boxed{120.5°}$

$\gamma_2 = 180° - 41.2° - 120.5° = \boxed{18.3°}$

$\dfrac{\sin 41.2°}{8.1} = \dfrac{\sin 18.3°}{c_2}$ $\quad c_2 = \dfrac{8.1 \sin 18.3°}{\sin 41.2°} = \boxed{3.9}$

3. $\beta = 75.3°$, $b = 12.4$, $c = 9.8$

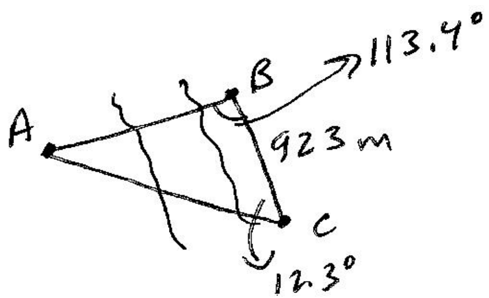
$b > c$ so, 1 triangle

$\dfrac{\sin 75.3°}{12.4} = \dfrac{\sin \gamma}{9.8}$ $\quad \gamma = \sin^{-1}\left(\dfrac{9.8 \sin 75.3°}{12.4}\right) = \boxed{49.9°}$

$\alpha = 180° - 75.3° - 49.9° = \boxed{54.8°}$

$\dfrac{\sin 75.3°}{12.4} = \dfrac{\sin 54.8°}{a}$ $\quad a = \dfrac{12.4 \sin 54.8°}{\sin 75.3°} = \boxed{10.5}$

4. $\overline{BC} = 923\text{ m}$, $\beta = 113.4°$, $\gamma = 12.5°$. Find $\overline{AB}$.



$\alpha = 180° - 113.4° - 12.3° = 54.1°$

$\dfrac{\sin 12.5°}{\overline{AB}} = \dfrac{\sin 54.1°}{923}$

$\overline{AB} = \dfrac{923 \sin 12.5°}{\sin 54.1°} = \boxed{246.6\text{ m}}$

5. $\gamma = 84.9°$, $a = 7.28$, $b = 8.51$

$c^2 = (7.28)^2 + (8.51)^2 - 2(7.28)(8.51)\cos 84.9°$

$c^2 = 114.404$

$\boxed{c = 10.7}$

$\dfrac{\sin 84.9°}{10.7} = \dfrac{\sin \alpha}{7.28}$ $\quad \alpha = \sin^{-1}\left(\dfrac{7.28 \sin 84.9°}{10.7}\right)$

$\alpha = \boxed{42.7°}$

$\beta = 180° - 84.9° - 42.7°$
$= \boxed{52.4°}$

6. $a = 6.2$, $b = 12.5$, $c = 13.8$

$(13.8)^2 = (6.2)^2 + (12.5)^2 - 2(6.2)(12.5)\cos \gamma$

$190.44 = 194.69 - 155\cos\gamma$
$-194.69 \quad -194.69$

$\dfrac{-4.25}{-155} = \dfrac{-155\cos\gamma}{-155}$

$\gamma = \cos^{-1}\left(\dfrac{4.25}{155}\right)$
$= \boxed{88.4°}$

$\dfrac{\sin 88.4°}{13.8} = \dfrac{\sin \alpha}{6.2}$

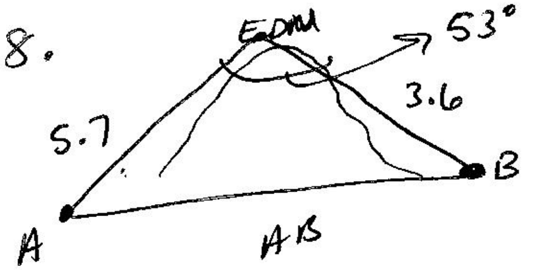
$\alpha = \sin^{-1}\left(\dfrac{6.2 \sin 88.4°}{13.8}\right)$
$= \boxed{26.7°}$

$\beta = 180° - 88.4° - 26.7°$
$= \boxed{64.9°}$

7. $\langle 4, -5 \rangle$

$\boxed{4i - 5j}$

8.



$\overline{AB}^2 = (5.7)^2 + (3.6)^2 - 2(5.7)(3.6)\cos 53°$

$\overline{AB}^2 = 20.75$

$\overline{AB} = \boxed{4.6 \text{ miles}}$

9. $\alpha = 15.0°$, $b = 10.7$, $c = 7.3$

$A = \frac{1}{2}(10.7)(7.3)\sin 15°$

$= \boxed{10.1 \text{ units squared}}$

10. $a = 73.5$, $b = 86.4$, $c = 34.9$

$$s = \frac{73.5 + 86.4 + 34.9}{2} = 97.4$$

$$A = \sqrt{(97.4)(97.4 - 73.5)(97.4 - 86.4)(97.4 - 34.9)} = \boxed{1265 \text{ units squared}}$$

11. $|v| = 38.6$, $\theta = 77.5°$

$V_x = 38.6 \cos 77.5° = \boxed{8.35}$

$V_y = 38.6 \sin 77.5° = \boxed{37.69}$

$\langle v_x, v_y \rangle$

$\boxed{\langle 8.35, 37.69 \rangle}$

12. $|v| = 20.6$, $\theta = 102.5°$

$V_x = 20.6 \cos 102.5° = -4.46$

$V_y = 20.6 \sin 102.5° = 20.11$

$\boxed{\langle -4.46, 20.11 \rangle}$

13. $\langle 8, -8\sqrt{3} \rangle$

$$|v| = \sqrt{(8)^2 + (-8\sqrt{3})^2} = \boxed{16}$$

$$\theta = \tan^{-1}\left(-\frac{8\sqrt{3}}{8}\right) = -60^\circ \rightarrow \boxed{300^\circ}$$

14. $3u - v \qquad u = \langle -1, 5 \rangle, \; v = \langle 4, -7 \rangle$

$$3\langle -1, 5 \rangle - \langle 4, -7 \rangle = \langle -3, 15 \rangle - \langle 4, -7 \rangle = \boxed{\langle -7, 22 \rangle}$$

15. $u \cdot v$

$$\langle -1, 5 \rangle \cdot \langle 4, -7 \rangle = (-1)(4) + (5)(-7) = -4 + -35 = \boxed{-39}$$

16. $\langle -1, 5 \rangle, \langle 2, 7 \rangle$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$$

$$u \cdot v = -2 + 35 = 33$$

$$|u| = \sqrt{(-1)^2 + (5)^2} = \sqrt{26}$$

$$|v| = \sqrt{(2)^2 + (7)^2} = \sqrt{53}$$

$$\theta = \cos^{-1}\left(\frac{33}{\sqrt{26 \cdot 53}}\right) = \boxed{27.3^\circ}$$

17. $\langle 2, -4 \rangle, \langle 6, 3 \rangle$

If $u \cdot v = 0$ then perpendicular

$$u \cdot v = (2)(6) + (-4)(3) = 12 - 12 = 0$$

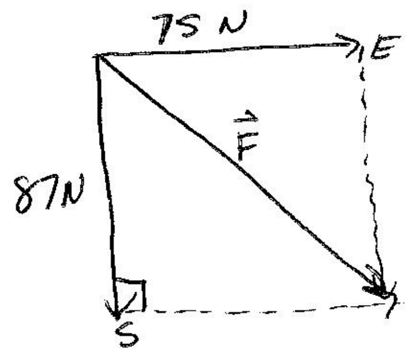
$\boxed{\text{perpendicular}}$

18. $\langle -1, 7 \rangle \; \langle 3, -21 \rangle$

$$\frac{7}{-1} \stackrel{?}{=} \frac{-21}{3}$$

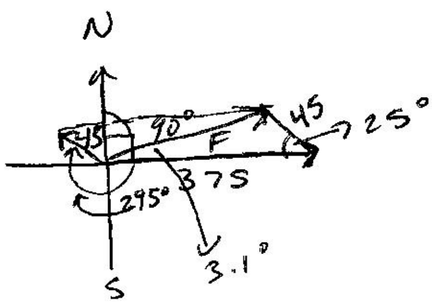
Yes congruent so, $\boxed{\text{parallel}}$

19.



$\vec{F} = \sqrt{(87)^2 + (75)^2} = \boxed{115\ N}$

20.



$\vec{F} = \sqrt{(375)^2 + (43)^2 - 2(375)(43)\cos 25°}$

$\vec{F} = \boxed{336.5\text{ mph - ground speed}}$

$\dfrac{\sin 25°}{336.5} = \dfrac{\sin A}{43}$

$A = \sin^{-1}\left(\dfrac{43 \sin 25°}{336.5}\right) = 3.1°$

Heading $90 - 3.1° = \boxed{86.9°}$